The World of Geometry

Geometry is the branch of mathematics that deals with the shape and size of objects. It not only provides us with a way to describe the world we can see and touch, but it allows us to understand how these objects interact, as well.

Before we can discuss the nature of geometrical figures, however, there are a number of terms we must define:

1. **Point** - a geometric object that has no dimension, only a location either in space or time.
2. **Line** - a collection of points along a straight path, with no endpoints. Points that lie along the same line are called **collinear**.
3. **Line segment** - any part of a line that has two definite endpoints.
4. **Ray** - any part of a line that has one definite endpoint.
5. **Angle** - two rays connected at a common endpoint.
6. **Plane** - a flat (two-dimensional) surface that extends endlessly in all directions. Points that lie on the same plane are called **coplanar**.
7. **Plane geometry** - the branch of geometry that deals with two-dimensional figures.
8. **Coordinate geometry** - the branch of geometry that deals with figures in a coordinate system such as a grid.
9. **Space (or spatial) geometry** - the branch of geometry that deals with three-dimensional figures.

Circles

One of the most common geometrical figures is the circle. A circle is defined as all points in the same plane that lie an equal distance from a central point; the points form a curve which, when traced, comes back around to its origin. The circle is named by its center point, and all points that lie inside the curve are called "**interior points**."

The distance from the center point to any point on the circle is called the **radius**, the plural of which is radii; the radius is therefore a line segment with one endpoint on the circle and the other at the center of the circle. In the following diagram, the radius is denoted by r.

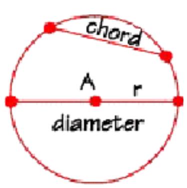


Figure 1: A circle. All points on the circle lie an equal distance from the central point, "A". "r" represents the radius of the circle; the diameter is twice the length of the radius. The chord is a line segment within a circle that connects any two points on the circle.

A **diameter** is a line segment that not only has both endpoints on the circle, but also passes through the center of that circle. A diameter is therefore twice the length of the radius, or 2r. This measurement can be represented mathematically as:

$$d = 2r$$

Two circles that have **congruent** [the same size] radii or diameters are called congruent circles.

A curved line that contains a portion of a circle is called an **arc**. Arcs can also be measured in degrees, like angles, and are classified in a similar way - minor arcs, major arcs, and semicircles. Since a circle contains 360 degrees (^{o}), the sum of all the arc angles in a circle must equal 360. A minor arc measures between 0 and 180 degrees; a major arc lies between 180 and 360 degrees; and a semicircle equals exactly 180. Arcs also have length, just as line segments do -- imagine placing a string around the circle, then picking it up, stretching it out and measuring it. The sum of all the lengths of the non-overlapping arcs of a circle is called the **circumference**, and it represents the distance around the circle.

Circles possess certain properties that make it easy to calculate relationships between the various parts. You may have heard the term [the Greek letter pi, pronounced pie]. This term represents the relationship of the circumference and the diameter of a circle, such that:

$$\Pi = \text{(the circumference) / (the diameter) of any circle} = 3.141592\ldots$$

This means that when you divide the circumference of any circle by its diameter, it will always be equal to 3.141592 . . .

By rearranging the terms, we find that:

$$\text{Circumference} = \text{Ð}(d) \text{ or, since the diameter} = 2 \text{ times the radius}$$

$$\text{Circumference} = \text{Ð}\ (2r)$$

Polygons

The word “polygon” comes from two Greek words -- “poly” meaning many, and “gon” meaning angle; polygons, therefore, are a class of geometrical figures that have many angles. They also have many sides, each of which is formed from a line segment. In fact, polygons are named according to the number of sides and angles they have. The most familiar polygons are the triangle (literally “three angles”), the rectangle, and the square (from another subclass, called quadrilaterals - see below). A regular polygon is one with all angles and sides congruent.

Triangles always lie in a single plane (that is, all their points are coplanar), and, like polygons, they can be classified either by their sides or their angles. By sides, triangles fall into 3 categories (see figure 2):

1. **Scalene** - a triangle with three sides of different lengths.
2. **Isosceles** - a triangle with two equal sides, called legs, and the third side called the base.
3. **Equilateral** - a triangle with three equal sides. In this type of triangle, the angles are also equal, so it can also be called an equiangular triangle.

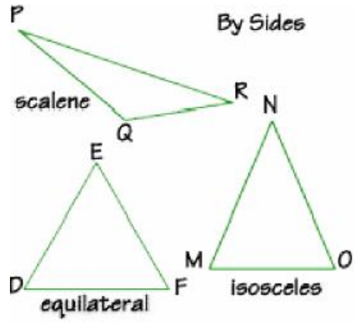
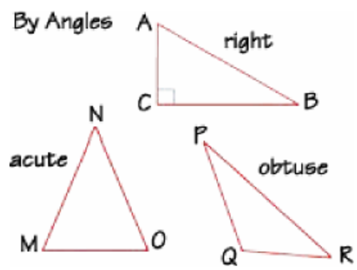


Figure 2: Shapes of triangles.

Triangles can be classified according to the length of their sides. A scalene triangle has three sides of different lengths. An isosceles triangle has two equal sides (called legs - line segments MN and NO), and a third side (called the base - line segment MO). An equilateral triangle has three equal sides.

Figure 3: Angles of triangles.

Triangles can also be classified according to the type(s) of angles they contain. An equilateral triangle has three equal angles of 60 degrees each. An acute triangle contains three angles each less than 90 degrees. An obtuse triangle has one angle larger than 90 degrees, and a right triangle has one angle exactly equal to 90 degrees.



Triangles can also be classified according to their angles (see figure 3). It is important to note that the sum of all the interior angles in a triangle must equal 180 degrees:

1. **Equilateral triangle** - a triangle where all angles are equal; each angle must, therefore, measure 60 degrees.
2. **Acute triangle** - a triangle with three acute angles, or three angles that each measure less than 90 degrees.
3. **Obtuse triangle** - a triangle with one angle that is greater than 90 degrees (angle PQR above).
4. **Right triangle** - a triangle where one of the angles measures exactly 90 degrees (a right angle). We draw a square at the angle to show it is a right angle (angle ACB above).

Quadrilaterals come in many different forms, but they all have several things in common: they have four sides (from "quad," meaning four); they are coplanar; they have two diagonals; and the sum of their four interior angles always equals 360 degrees (see figure 4).

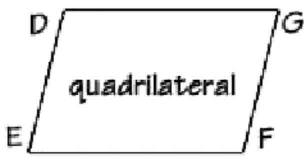


Figure 4: A quadrilateral.

A parallelogram, a subclass of quadrilateral that not only is coplanar, has four sides, and interior angles equal to 360 degrees, but also has two parallel pairs of opposite sides.

Quadrilaterals come in numerous forms, each of which has its own special properties:

1. **Parallelogram** - a quadrilateral with two parallel pairs of opposite sides.
2. **Rectangle** - a quadrilateral with two parallel pairs of opposite sides parallel, as well as four right angles. It is therefore a special type of parallelogram.
3. **Square** - a quadrilateral with two pairs of parallel sides, four right angles, and all four sides equal. It is also both a rectangle and a parallelogram.
4. **Rhombus** - a parallelogram with four equal sides. It is not always a rectangle, however, because it does not have to have four right angles.
5. **Trapezoid** - a quadrilateral with only one pair of parallel sides. It is therefore not a parallelogram.
6. **Kite** - a quadrilateral with two pairs of adjacent sides that are equal.

Activity

1. Draw several circles of various sizes (using a compass will help you draw them more accurately). Measure both the radius (or diameter) and circumference of each circle and, using the formula provided above, calculate the value of . I it the same for every circle?
2. MORE ADVANCED: The Pythagorean Theorem (a mathematical principle developed by the Greek mathematician Pythagorus) tells us that in a right triangle the sum of the squares of each of the legs (AC and BC in the diagram above) is equal to the square of the hypotenuse (the side opposite the right angle). In other words:

$$a_2 + b^2 = c_2$$

If the legs are equal to 3 and 4, what is the value of the hypotenuse?